NEW FORMAT

AJ Sadler

Mathematics Methods

1

Student Book

Unit 1

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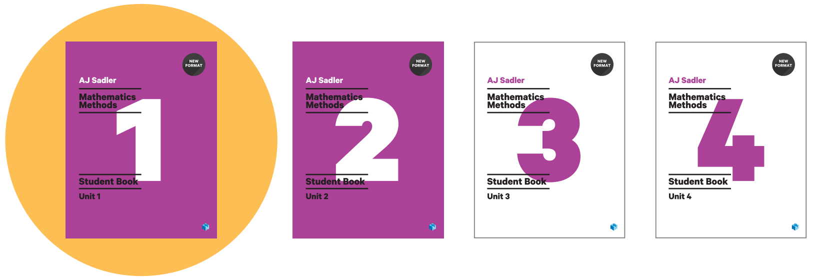
PREFACE



This text targets Unit One of the West Australian course *Mathematics Methods*, a course that is organised into four units altogether, units one and two for year eleven and units three and four for year twelve.

The West Australian course, *Mathematics Methods*, is based on the Australian Curriculum Senior Secondary course *Mathematical Methods*. At the time of writing there is very little difference between the content of Unit One of the two courses, so this text would also be suitable for anyone following Unit One of the Australian Curriculum course, *Mathematical Methods*.

The book contains text, examples and exercises containing many carefully graded questions. A student who studies the appropriate text and relevant examples should make good progress with the exercise that follows.

The book commences with a section entitled **Preliminary work** to give the reader an early reminder of some of the work from earlier years that it will be assumed readers are familiar with, or for which the brief outline included in the section may be sufficient to bring the understanding of the concept up to the necessary level.

As students progress through the book they will encounter questions involving this preliminary work in the **Miscellaneous Exercises** that feature at the end of each chapter. These miscellaneous exercises also include questions involving work from preceding chapters to encourage the continual revision needed throughout the unit.

Students should also find that the content in some chapters involves work encountered in previous years, thus allowing speedy progress through those sections.

Some chapters commence with a '**Situation**' or two for students to consider, either individually or as a group. In this way students are encouraged to think and discuss a situation, which they are able to tackle using their existing knowledge, but which acts as a fore-runner and stimulus for the ideas that follow. Students should be encouraged to discuss their solutions and answers to these situations and perhaps to present their method of solution to others. For this reason answers to these situations are generally not included in the book.

At times in this series of books I have found it appropriate to go a little outside the confines of the syllabus for the unit involved. In this regard readers will find in this text that the Pythagorean theorem is assumed and used, the distance between two points with known coordinates is covered and the trigonometric identity $\sin^2 A + \cos^2 A = 1$ is included. When considering $y = a \sin x$, $y = \sin bx$ and $y = \sin (x - c)$, I also consider the more general $y = a \sin [b(x - c)] + d$.

The sine and cosine rules are considered early in this text so that students studying *Mathematics Specialist* cover this work in good time for its use in that course.

Alan Sadler

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IMPORTANT NOTE

This series of texts has been written based on my interpretation of the appropriate *Mathematics Methods* syllabus documents as they stand at the time of writing. It is likely that as time progresses some points of interpretation will become clarified and perhaps even some changes could be made to the original syllabus. I urge teachers of the *Mathematics Methods* course, and students following the course, to check with the appropriate curriculum authority to make themselves aware of the latest version of the syllabus current at the time they are studying the course.

Acknowledgements

As with all of my previous books I am again indebted to my wife, Rosemary, for her assistance, encouragement and help at every stage.

To my three beautiful daughters, Rosalyn, Jennifer and Donelle, thank you for the continued understanding you show when I am "still doing sums" and for the love and belief you show.

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PRELIMINARY WORK

This book assumes that you are already familiar with a number of mathematical ideas from your mathematical studies in earlier years.

This section outlines the ideas which are of particular relevance to Unit One of the *Mathematics Methods* course and for which some familiarity will be assumed, or for which the brief explanation given here may be sufficient to bring your understanding of the concept up to the necessary level.

Read this 'preliminary work' section and if anything is not familiar to you, and you don't understand the brief explanation given here, you may need to do some further reading to bring your understanding of those concepts up to an appropriate level for this unit. (If you do understand the work but feel somewhat 'rusty' with regards to applying the ideas some of the chapters afford further opportunities for revision, as do some of the questions in the miscellaneous exercises at the end of chapters.)

- Chapters in this book will continue some of the topics from this preliminary work by building on the assumed familiarity with the work.
- The **miscellaneous exercises** that feature at the end of each chapter may include questions requiring an understanding of the topics briefly explained here.

Types of number

It is assumed that you are already familiar with counting numbers, whole numbers, integers, factors, multiples, prime numbers, composite numbers, square numbers, negative numbers, fractions, decimals, the rule of order, percentages, rounding to particular numbers of decimal places, truncating, the square root and the cube root of a number, powers of numbers (including zero and negative powers), and can use this familiarity appropriately. An ability to simplify simple expressions involving square roots is also assumed.

e.g. $\sqrt{8} = \sqrt{4 \times 2}$
$= 2\sqrt{2}$

$\sqrt{27} + \sqrt{75} = \sqrt{9 \times 3} + \sqrt{25 \times 3}$
$= 3\sqrt{3} + 5\sqrt{3}$
$= 8\sqrt{3}$

$\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{3\sqrt{2}}{2}$

An understanding of numbers expressed in *standard form* or *scientific notation*, e.g. writing 260 000 in the form 2.6×10^5 or writing 0.0015 in the form 1.5×10^{-3}, is also assumed.

The set of numbers that you are currently familiar with is called the set of **real numbers**. We use the symbol $\mathbb{R}$ (or **R**) for this set. Sets like the whole numbers, the integers, the primes, the square numbers etc. are each subsets of $\mathbb{R}$.

Direct proportion

If one copy of a particular book weighs 1.5 kg we would expect two copies of this book to weigh 3 kg, three to weigh 4.5 kg, four to weigh 6 kg, five to weigh 7.5 kg and so on. Every time we increase the number of books by one the weight goes up by 1.5 kg. Hence the straight line nature of the graph of this situation shown below.

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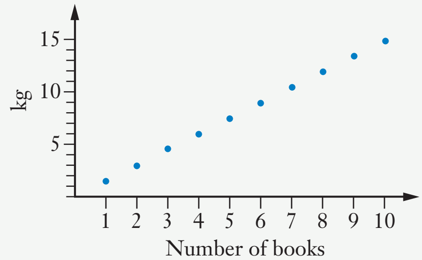


The number of books and the weight of the books are in **direct proportion** (also called **direct variation**):

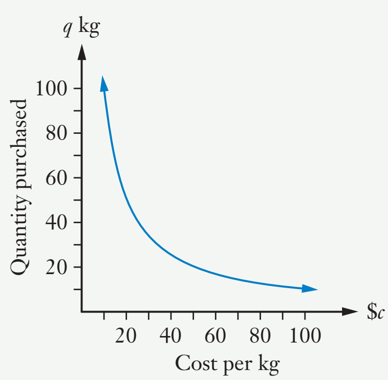
For two quantities that are in direct proportion, as one quantity is multiplied by a certain number then the other quantity is also multiplied by that number.

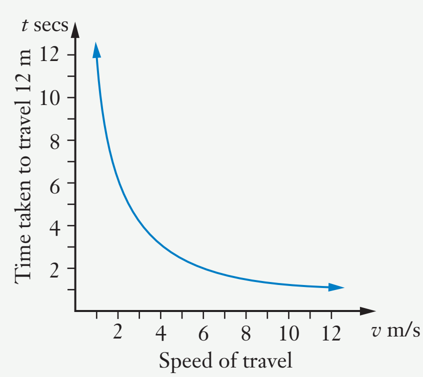
- Doubling one will cause the other to double.
- Halving one will cause the other to halve.
- Trebling one will cause the other to treble. Etc.

Inverse proportion

The graph below left shows the amount, q kg, of a particular commodity that could be purchased for \$1000 when the commodity costs \$$c$ per kg.

The graph below right shows the time taken, t seconds, to travel a distance of 12 metres, by something travelling at v metres/second.





Rule: $q = \dfrac{1000}{c}$ $\qquad t = \dfrac{12}{v}$

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Each of the two previous situations involve **inverse proportion**:

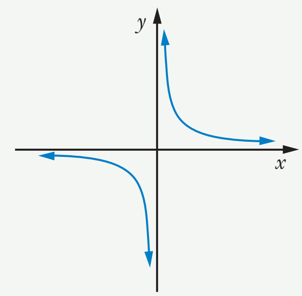
Double the cost of the commodity → Divide the quantity purchased by 2.
Treble the cost of the commodity → Divide the quantity purchased by 3. Etc.

Double the speed of travel → Divide the time taken by 2.
Treble the speed of travel → Divide the time taken by 3. Etc.

If two variables are inversely proportional to each other the graph of the relationship will be that of a **reciprocal function**.

If x and y are inversely proportional the relationship will have an equation of the form

$$y = \frac{k}{x}.$$



The graph will have the characteristic shape of a reciprocal relationship, as shown on the right, though in many applications negative values for the variables may not make sense and only that part of the typical shape for which both variables are positive will apply, as in the two earlier examples.

Use of algebra

WS Expanding algebraic expressions

WS Factorising algebraic expressions

It is assumed that you are already familiar with manipulating algebraic expressions, in particular:

- **Expanding and simplifying**:

For example	$4(x + 3) - 3(x + 2)$	expands to	$4x + 12 - 3x - 6$
		which simplifies to	$x + 6$
	$(x - 7)(x + 1)$	expands to	$x^2 + 1x - 7x - 7$
		which simplifies to	$x^2 - 6x - 7$

- **Factorising**:

For example,	$21x + 7$	factorises to	$7(3x + 1)$
	$15apy + 12pyz - 6apq$	factorises to	$3p(5ay + 4yz - 2aq)$
	$x^2 - 6x - 7$	factorises to	$(x - 7)(x + 1)$
	$x^2 - 9$	factorises to	$(x - 3)(x + 3)$

the last one being an example of the *difference of two squares* result:

$x^2 - y^2$ factorises to $(x - y)(x + y)$.

You should also be familiar with the idea that solving an equation involves finding the value(s) the unknown can take that make the equation true.

For example, $x = 5.5$ is the solution to the equation $15 - 2x = 4$
because $15 - 2(5.5) = 4$.

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Similarly, given two equations in two unknowns, for example: $\begin{cases} 5x - 2y = 6 \\ 3x + 2y = 26 \end{cases}$

'solving' means finding the pair of values that satisfy both equations, in this case the values are $x = 4$ and $y = 7$.

It is anticipated that you are familiar with solving various types of equation. For example:

Linear equations:

$3x - 5 = 7$	$6x + 1 = 15 - x$	$3(2x - 1) - (5 - 2x) = -20$
Solution: $x = 4$	Solution: $x = 2$	Solution: $x = -1.5$

Equations with fractions:

$$\frac{3x-5}{2} = 8 \qquad \text{Solution: } x = 7$$

$$\frac{21-x}{2x+3} = 2 \qquad \text{Solution: } x = 3$$

Simultaneous equations:

e.g. $\begin{cases} y = 2x - 1 \\ y = x + 2 \end{cases}$ Solutions: $x = 3, y = 5$

e.g. $\begin{cases} 2x + y = 9 \\ 2x - 3y = 13 \end{cases}$ $x = 5, y = -1$

WS Simultaneous equations

Quadratic equations (including ones that are readily factorised):

e.g. $x^2 = 25$ Solutions: $x = \pm 5$
e.g. $(2x - 3)(x + 1) = 0$ Solutions: $x = 1.5, \quad x = -1$
e.g. $x^2 - x - 20 = 0$
i.e. $(x - 5)(x + 4) = 0$ Solutions: $x = 5, \quad x = -4$

Factorised cubics:

e.g. $x(x + 1)(x - 5) = 0$ Solutions: $x = 0, \quad x = -1, \quad x = 5$

Pascal's triangle

With an understanding of powers and an ability to expand brackets and to collect like terms we can show that:

$(x + y)^0 = 1$

$(x + y)^1 = x + y$

$(x + y)^2 = (x + y)(x + y) = x^2 + 2xy + y^2$

$(x + y)^3 = (x + y)(x + y)(x + y) = x^3 + 3x^2y + 3xy^2 + y^3$

$(x + y)^4 = (x + y)(x + y)(x + y)(x + y) = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x + y)^5 = (x + y)(x + y)(x + y)(x + y)(x + y) = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Noticing how the first six lines of **Pascal's triangle**, shown on the right, feature in the above expansions of $(x + y)^n$ for $n = 0, 1, 2, 3, 4$ and 5, allows us, by using the appropriate lines of Pascal's triangle, to write down expansions of $(x + y)^n$ for higher values of n directly. For example:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

$(x + y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

$(x + y)^8 = x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$

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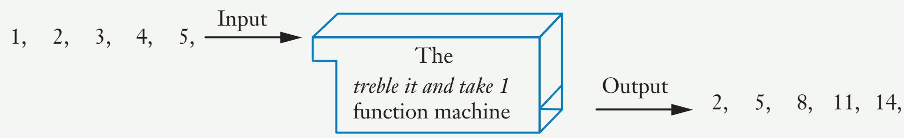
Function notation

Given the rule $y = 3x - 1$ and a particular value of x, say 5, we can determine the corresponding value of y, in this case 14.

The rule performs the function '*treble it and take one*' on any given x value and outputs the corresponding y value.

It can be helpful to consider a function as a machine with a specific output for each given input:



In mathematics any rule that takes any given input value and assigns to it a particular output value is called a **function**.

We can write functions using the notation $f(x)$, pronounced 'f of x'.

For the 'treble it and take one' function we write $\quad f(x) = 3x - 1.$

For this function:

$$\begin{aligned} f(1) &= 3(1) - 1 = 2 \\ f(2) &= 3(2) - 1 \\ &= 5 \text{ etc.} \end{aligned}$$

Alternatively we could use a second variable, say y, and express the rule as

$$y = 3x - 1.$$

The value of the variable y depends on the value chosen for x. We call y the **dependent variable** and x the **independent variable**. The dependent variable is usually by itself on one side of the equation whilst the independent variable is 'wrapped up' in an expression on the other side.

Types of function

From your mathematical studies of earlier years you should be familiar with:

1 Linear functions

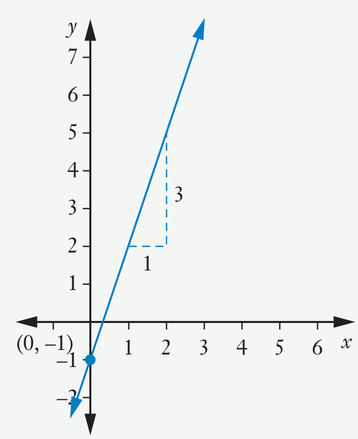
These have:

- **equations** of the form $y = mx + c$.
  For example: $y = 3x - 1$ for which $m = 3$ and $c = -1$.
- **tables of values** for which each unit increase in the x values sees a constant increase of m in the corresponding y values
  For example, for $y = 3x - 1$

x	0	1	2	3	4	5	6
y	–1	2	5	8	11	14	17

3 3 3 3 3 3

- **graphs** that are straight lines with gradient m and cutting the vertical axis at the point with coordinates $(0, c)$.
  For example, for $y = 3x - 1$



II Quadratic functions

These have

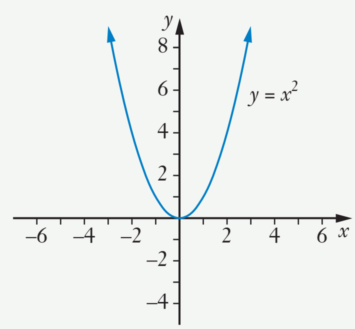
- **equations** of the form $y = ax^2 + bx + c, a \neq 0$.
  For example, with $a = 1, b = 0$ and $c = 0$
  we have $y = x^2$ the most basic quadratic.
  With $a = 2, b = -6$ and $c = 1$
  we have $y = 2x^2 - 6x + 1$.
  The equations of quadratic functions are sometimes written in the
  alternative forms $y = a(x - p)^2 + q$
  and $y = a(x - d)(x - e)$.
- **tables of values** with a constant *second difference* pattern
  For example, for $y = 2x^2 - 6x + 1$

x	0	1	2	3	4	5	6
y	1	–3	–3	1	9	21	37

1st diff –4 0 4 8 12 16

2nd diff 4 4 4 4 4

- **graphs** that are the same basic shape as that of $y = x^2$ shown on the right, but that may be moved left, right, up, down, flipped over, squeezed or stretched.



III Reciprocal functions

- As mentioned a few pages earlier when we were considering inverse proportion, reciprocal functions have **equations** of the form

$$y = \frac{k}{x}. \qquad \text{(Undefined for } x = 0\text{)}$$

For example, with $k = 12$, $y = \dfrac{12}{x}$

- **tables of values** for which the x and y paired values have a common product (equal to k) for example, for $y = \dfrac{12}{x}$:

x	–4	–3	–2	–1	0	1	2	3	4
y	–3	–4	–6	–12	Undefined	12	6	4	3
	$-4 \times -3 = 12$	$-3 \times -4 = 12$	$-2 \times -6 = 12$	$-1 \times -12 = 12$		$1 \times 12 = 12$	$2 \times 6 = 12$	$3 \times 4 = 12$	$4 \times 3 = 12$

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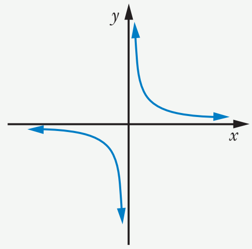
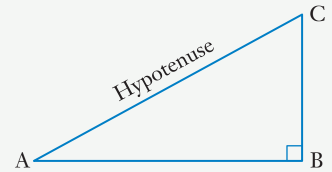
- **graphs** with the characteristic shape shown on the right, reflected in the y-axis if the k in $y = \frac{k}{x}$ is negative.



Pythagoras and trigonometry

It is anticipated that you have already encountered the Pythagorean theorem and the trigonometrical ratios of sine, cosine and tangent.



A very brief revision of the terminology and basic facts is included here.

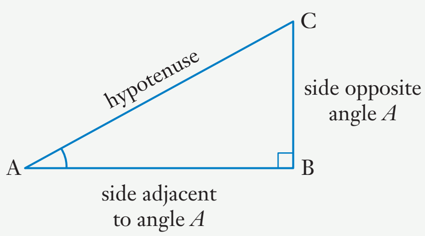
In a right triangle we call the side opposite the right angle the **hypotenuse**.

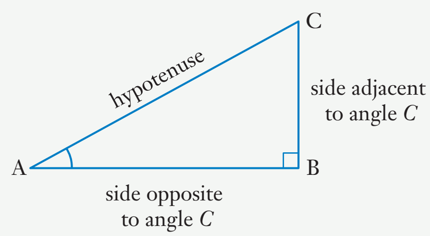
The Pythagortean theorem states that:

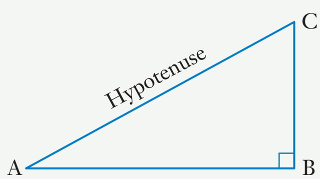
The square of the length of the hypotenuse of a right angled triangle is equal to the sum of the squares of the lengths of the other two sides.

Thus, for the triangle shown, $AC^2 = AB^2 + BC^2$.

We refer to the other two sides of a right triangle as being **opposite** or **adjacent** to (next to) particular angles of the triangle.







We then define the sine, cosine and tangent ratios as follows:

$$\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{CB}{AC} \qquad \sin C = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC} \qquad \cos C = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{CB}{AC}$$

$$\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{CB}{AB} \qquad \tan C = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{AB}{CB}$$

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The sine, cosine and tangent ratios can be remembered using the mnemonic **SOHCAHTOA**:

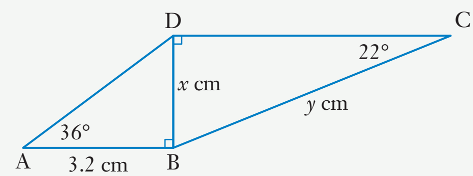
SOH	CAH	TOA
$\text{Sin} = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$	$\text{Cos} = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$	$\text{Tan} = \dfrac{\text{Opposite}}{\text{Adjacent}}$

Notice that it then follows that

$$\begin{aligned}\frac{\sin x}{\cos x} &= \frac{\text{Opposite}}{\text{Hypotenuse}} \div \frac{\text{Adjacent}}{\text{Hypotenuse}}\\ &= \frac{\text{Opposite}}{\text{Adjacent}}\\ &= \tan x\end{aligned}$$

The trigonometrical ratios of sine, cosine and tangent and the theorem of Pythagoras allow us to determine the lengths of sides and sizes of angles of right triangles, given sufficient information.

For example, given the diagram on the right, x and y can be determined as shown below:



From △ABD $\qquad \tan 36° = \dfrac{x}{3.2}$

$\therefore \qquad x = 3.2 \tan 36°$

$\qquad \approx 2.32$

Correct to 1 decimal place, $x = 2.3$.

From △BCD $\qquad \sin 22° = \dfrac{x}{y}$

Hence $\qquad y \sin 22° = x$

and so $\qquad y = \dfrac{x}{\sin 22°}$

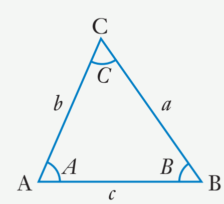
```
3.2 × tan36
                2.32493609
Ans ÷ sin22
               6.206340546
```

Being sure to use the accurate value of x, not the rounded value of 2.3, we obtain

$y \approx 6.21.$

Correct to 1 decimal place, $y = 6.2$.

With the usual convention for labelling a triangle, i.e. the angles use the capital letter of the vertex and lower case letters are used for sides opposite each angle, you may also be familiar with the following rules for △ABC:



Area of a triangle: $\qquad \dfrac{ab\sin C}{2}$

The sine rule: $\qquad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

The cosine rule: $\qquad a^2 = b^2 + c^2 - 2bc\cos A$

These rules will be revised in this book in chapter 1, *Trigonometry*.

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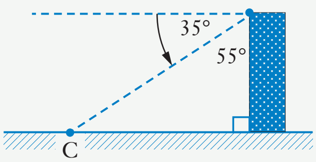
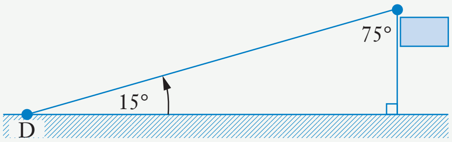
An understanding of the use of **bearings** to indicate direction and of the concepts of an **angle of elevation** (from the horizontal, up) and **an angle of depression** (from the horizontal, down) is also assumed.

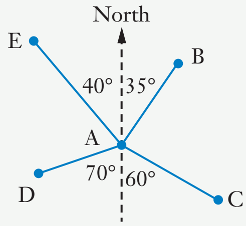
Thus in the diagram below left the angle of depression of point *C* from the top of the tower is 35° and in the diagram below right the angle of elevation of the top of the flagpole from point *D* is 15°.





In the diagram on the right, from point A, the bearings of points B, C, D and E are shown in the table.

Point	Three-figure bearing	Compass bearing
B	035°	N 35° E
C	120°	S 60° E
D	250°	S 70° W
E	320°	N 40° W



Accuracy and trigonometry questions

Note that on the previous page the accurate value of x was used to determine y, not the rounded value, thus avoiding errors caused by premature rounding.

Note also that the answers for x and y were given as rounded values. Sometimes a situation may stipulate the degree to which answers must be rounded but if that is not the case you should round 'appropriately'. Just what is appropriate depends upon the accuracy of the data given and the situation. For example, in the calculation of the previous page it would be inappropriate to claim our value for x as 2.324 936 09, the answer obtained from a calculator, because it is far beyond the accuracy of the information used to obtain it, i.e. 3.2 cm and 36°.

In general our final answer should not be more accurate than the accuracy of the data we use to obtain it. The question gave us a length in cm, to 1 decimal place, so we should not claim greater accuracy for lengths we determine. Sometimes we may need to use our judgement of the likely accuracy of the given data. Given a length of 5 cm we might assume this has been measured to the nearest mm and hence give answers similarly to the nearest mm. (In theory a measurement of 5 cm measured to the nearest mm should be recorded as 5.0 cm but this is often not done.)

If the nature of the situation is known we might be able to judge the appropriate level of rounding. For example if asked to determine the dimensions of a metal plate that is to be made and then inserted into a patient, the level of accuracy may need to be greater than if dimensions were needed for some other situations.

In situations where accuracy is crucial any given measurements could be given with 'margins of error' included, for example 3.2 cm ± 0.05 cm, 36° ± 0.5°. More detailed error analysis could then be carried out and the margins of error for the answer calculated. However this is beyond the scope of this text.

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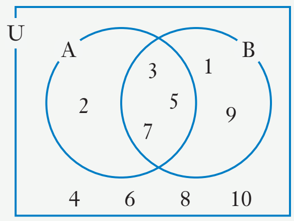
Sets



The **Venn diagram** on the right shows the **universal set**, U, which contains all of the **elements** currently under consideration, and the sets A and B contained within it.

We use 'curly brackets' to list a set. Thus

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{2, 3, 5, 7\} \text{ and } B = \{1, 3, 5, 7, 9\}$$

Set A has 4 members or elements.

We write $n(A) = 4$ or $|A| = 4$.

The number 9 is a member of B. We write $9 \in B$.

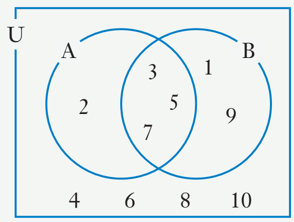
The number 2 is not in set B. We write $2 \notin B$.

$\{2, 7\}$ is a **subset** of A. We write $\{2, 7\} \subset A$.

The order that we list the elements of a set is unimportant. The set {a, b, f } is the same set even if we list the three letters in a different order.

If a set has no elements it is said to be empty. We use $\varnothing$ as the symbol for an empty set. For example, {multiples of 4 that are odd numbers} $= \varnothing$.

If a set has an infinite number of members we say it is an infinite set. For example the positive integers form an infinite set, $\{1, 2, 3, 4, 5, 6, \ldots\}$, as indicated by the '…'.



We use the symbol '$\cap$' for the overlap or **intersection** of two sets.

Thus $A \cap B = \{3, 5, 7\}$

We use '$\cup$' for the **union** of two sets.

Thus $A \cup B = \{1, 2, 3, 5, 7, 9\}$

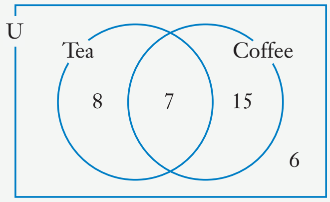
We use A' or $\bar{A}$ for the **complement** of A, i.e. everything in the universal set that is not in A. Thus $A' = \{1, 4, 6, 8, 9, 10\}$.

Venn diagrams can also provide a method for displaying information about the *number of elements* in various sets and can help to solve problems involving information of this kind. For example, suppose that 36 office workers were asked whether they had drunk tea or coffee during their morning breaks in the previous two weeks and further suppose that:

7 said they had drunk both,

8 said they had drunk only tea

and 6 said they had drunk neither.



This information could be used to create the Venn diagram shown on the right.

Asked how many had drunk coffee we can see from the Venn diagram that 22 had drunk coffee.

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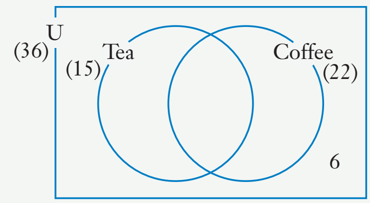
In some questions of this type the information is not supplied in a 'nice' order and we may need to read on through the information before being able to accurately place a number in its space on the Venn diagram.

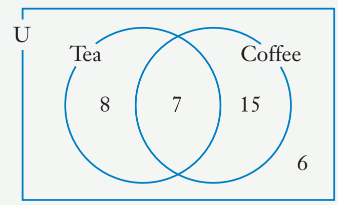
For example, in the previous tea/coffee situation suppose information had been presented as follows:

> *A survey involving 36 office workers asked whether they had drunk tea or coffee during their morning breaks in the previous two weeks. The survey found that*
>
> *15 had drunk tea,*
>
> *22 had drunk coffee*
>
> *and 6 said they had drunk neither.*

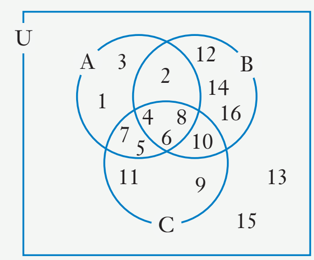
As we read the information we can 'note' it on the diagram, as shown by the bracketed numbers in the Venn diagram on the right. However we cannot accurately place numbers in the appropriate spaces until we reach the last piece of information, i.e. 6 said they had drunk neither.



Only then can we complete the Venn diagram as shown.



The Venn diagram on the right shows three sets A, B and C and the universal set U.



In this case

$$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$$
$$A \cap B = \{2, 4, 6, 8\}$$
$$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$$
$$A \cap (B \cup C) = \{2, 4, 5, 6, 7, 8\}$$
$$A \cap B \cap C = \{4, 6, 8\}$$
$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16\}$$

Why is it that we can write $A \cap B \cap C$ and not have to write this as $A \cap (B \cap C)$ or perhaps $(A \cap B) \cap C$? Why could we not do this with $A \cap (B \cup C)$?

Similarly why is it that we can write $A \cup B \cup C$ and not have to consider $A \cup (B \cup C)$ or $(A \cup B) \cup C$?

Probability

Note: The exercises in chapter 9, *Sets and probability*, contain questions that allow practice in some of the ideas about probability briefly explained here.

The probability of something happening is a measure of the likelihood of it happening and this measure is given as a number between zero (no chance of happening) to 1 (certain to happen).

In some cases we determine the probability of an event occurring by observing the outcome of a repeated number of trials in which the event is a possibility. The **long term relative frequency** with which the event occurs is then our best guess at the probability of the event occurring. Further trials may cause us to adjust this suggested probability.

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For example if we flipped a biased coin five hundred times and found that it landed tail uppermost on 400 of these occasions we would suggest that in any one flip:

The probability that the coin lands with the tail uppermost $= \frac{400}{500} = \frac{4}{5}$.

This could also be expressed as a decimal, 0.8, or percentage, 80%.

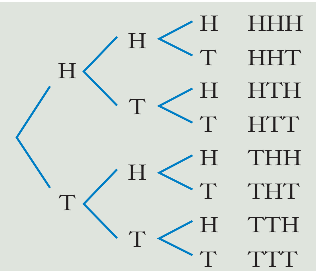
Probability based on observed data like this is called **empirical probability**.

Activities such as rolling a die or flipping a coin are examples of **random phenomenon**. We are unable to consistently predict the outcome of a particular die roll or coin flip but when these activities are repeated a large number of times each has a predictable long run pattern.

The list of all possible outcomes that can occur when something is carried out is called the **sample space**. For example, for one roll of a normal fair die the sample space is: 1, 2, 3, 4, 5, 6.

The probability of an event occurring can be determined without the need for repeated experiment if we are able to present the sample space as a list of **equally likely outcomes**. We can then find a theoretical probability rather than an empirical probability.

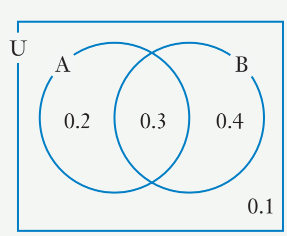
Three common ways of presenting the equally likely outcomes are shown below.

<table>
<tr><th>List</th><th>Table</th><th>Tree diagram</th></tr>
<tr><td>Rolling a normal die once.</td><td>Roll two normal dice and sum the numbers.</td><td>Flip a coin three times and note outcome.</td></tr>
<tr><td>6 equally likely outcomes.</td><td>36 equally likely outcomes.</td><td>8 equally likely outcomes.</td></tr>
<tr><td>1, 2, 3, 4, 5, 6.</td><td>
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr>
<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
</table>
</td><td>
</td></tr>
</table>

If we roll a normal die once the probability of getting a 3 is one-sixth. We write this as: $P(3) = \frac{1}{6}$.

An event occurring and it not occurring are said to be **complementary events**. If the probability of an event occurring is 'a' then the probability of it not occurring is $1 - a$.

A Venn diagram may be used as a way of presenting the probability of events A and/or B occurring, as shown on the right.



In this case:

$$P(A) = 0.2 + 0.3 \\ = 0.5$$

$$P(B) = 0.3 + 0.4 \\ = 0.7$$

$$P(\bar{A}) = 0.4 + 0.1 \\ = 0.5$$

$$P(A \cup B) = 0.2 + 0.3 + 0.4 \\ = 0.9$$

$$P(A \cap B) = 0.3$$

$$P(U) = 0.2 + 0.3 + 0.4 + 0.1 \\ = 1, \text{ as we would expect.}$$

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